PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Fall Semester 2011**

College of Sciences

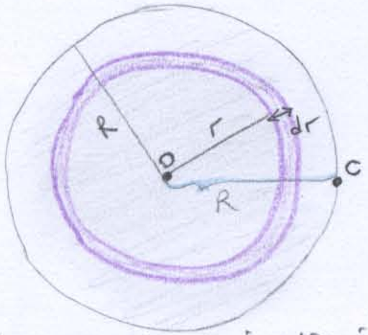
Section 1 **Quiz 9** **1 December 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

Find the moment of inertia of a uniform circular disk, of mass M and radius R, about an axis perpendicular to the disk at its edge, at distance R from the center of the disk.

We calculate I_c from I_o using the parallel-axis theorem



let's find I_o, rotating about an axis perpendicular to the disk at its center.

The moment of inertia is $I = \int r^2 dm = \int dI$

$dm = \rho dA$, Mass element has an area as shown with radius r and thickness dr

$dA = 2\pi r dr$ $\quad \rho = \frac{M}{A} = \frac{M}{\pi R^2} = \frac{dm}{dA}$, ρ : mass per unit volume

$$\Rightarrow I_o = \int_0^R r^2 \cdot \rho \cdot 2\pi r dr = 2\pi\rho \int_0^R r^3 dr = 2\pi\rho \left[\frac{r^4}{4}\right]_0^R = 2\pi\rho \frac{R^4}{4}$$

$$\Rightarrow I_o = 2\pi \cdot \frac{M}{\pi R^2} \cdot \frac{R^4}{4} = \frac{1}{2} MR^2$$

Check from Table 9.2, case (f)

$$\Rightarrow I_c = I_o + MR^2$$

(parallel axis theorem)

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Section 2 **Quiz 9** **1 December 2011**

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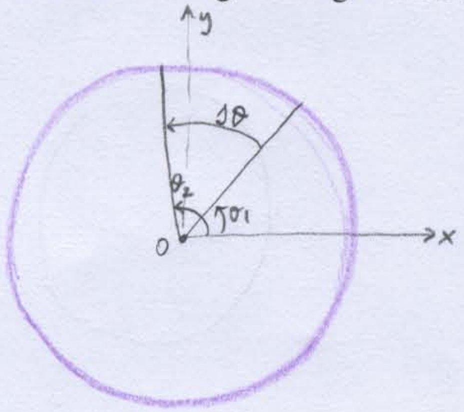
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

A disk of mass M and radius R is rotating about a central perpendicular axis. Its initial angular velocity ω_0 is doubled after a time τ. Assuming constant acceleration determine the angle through which the disk rotates in τ.



If the angular acceleration is constant, then θ, ω_z and α_z are related by simple kinematic equations analogous to those for straight-line motion with constant linear acceleration.

$$\omega_z = \omega_{0z} + \alpha_z \cdot t \quad \Rightarrow \quad \omega_{0z} = \alpha_z \cdot \tau \quad \Rightarrow \alpha_z = \frac{\omega_{0z}}{\tau}, \text{ constant}$$

$$\Rightarrow \theta - \theta_0 = \Delta\theta = \frac{1}{2}(\omega_{0z} + \omega_z)\cdot\tau \quad \Rightarrow \Delta\theta = \frac{1}{2}(3\omega_{0z})\cdot\tau \; //$$

or

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z \cdot \Delta\theta \quad \Rightarrow 4\omega_{0z}^2 = \omega_{0z}^2 + 2\cdot\left(\frac{\omega_{0z}}{\tau}\right)\cdot\Delta\theta$$

$$\Delta\theta = \frac{3\omega_{0z}^2 \cdot \tau}{2\omega_{0z}} = \frac{3}{2}\omega_{0z}\cdot\tau \; //$$

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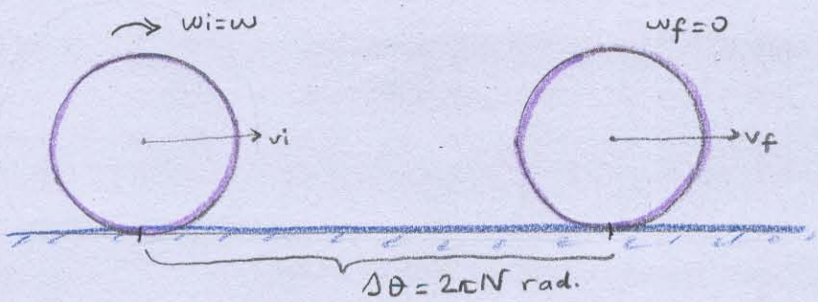
Section 3 Quiz 9 1 December 2011

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name: Student ID: Signature:

How much average power is dissipated when a wheel of radius R and mass M is brought to rest in N revolutions, with a constant angular acceleration, from an angular initial velocity ω?



$v = R\omega$
(We assume that there is a rotation without slipping)

$$\Delta W = \Delta KE_T = \Delta KE_{rot.} + \Delta KE_{translation}$$
$$= \left(\tfrac{1}{2} I \cancel{\omega_f^2} - \tfrac{1}{2} I \omega_i^2\right) + \left(\tfrac{1}{2} M \cancel{v_f^2} - \tfrac{1}{2} M v_i^2\right)$$
$$= -\tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 + \left(-\tfrac{1}{2}M(R\omega)^2\right)$$
$$\Rightarrow \Delta W = \Delta KE = -\tfrac{3}{4}MR^2\omega^2$$

where $I = \frac{1}{2}MR^2$ (Table 9.2 case (f)). We take $\Delta U = 0$

Let's find Δt

$$\theta - \theta_0 = \tfrac{1}{2}(\omega_i + \omega_f)\cdot\Delta t \qquad (\text{since } \alpha \text{ constant})$$
$$\Rightarrow 2\pi N = \tfrac{1}{2}\omega \cdot \Delta t$$
$$\Rightarrow \Delta t = \frac{4\pi N}{\omega}$$

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{-\frac{3}{4}MR^2\omega^2}{4\pi N/\omega} = -\frac{3MR^2\omega^3}{16\pi N}$$

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Section 4 **Quiz 9** **1 December 2011**

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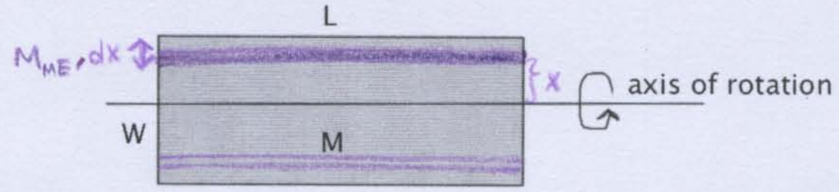
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

Find the moment of inertia of a thin rectangular uniform sheet of metal, of mass M, length L and width w, about the axis shown in the figure.



Check the solutions of the Quizes of the last years (2007, 2008, ...).

Let's calculate the moment of inertia I of a uniform solid infinitely thin sheet of metal with mass M_{ME}, as a mass element, at a distance x from the rotating axis.

$$I_{ME} = I_{\substack{Mass\\Element}} = \int x^2\, dm$$

As the rod is infinitely thin, every point on it has the same distance to the rotating axis.

$$I_{M.E} = \int x^2\, dm = x^2 \int dm = M_{\substack{mass\\Element}}\, x^2$$

$$M_{\substack{mass\\Element}} = \frac{M}{w.L}\cdot L\cdot dx = \rho\cdot L\cdot dx \quad \Longleftarrow \quad \rho = \frac{M}{w.L} = \frac{M_{m.E}}{L.dx}$$

Then integrate the infinitely thin rod to find the total moment of inertia.

$$I = \int I_{ME} = \int_{-w/2}^{w/2} x^2\cdot\rho\cdot L\cdot dx = \rho\cdot L\left[\frac{x^3}{3}\right]_{-w/2}^{w/2} = \frac{\rho L}{3}\left[\frac{w^3}{8} - \left(-\frac{w^3}{8}\right)\right]$$

$$I = \frac{\rho\cdot L}{3}\,\frac{w^3}{4} = \frac{M}{w\cancel{L}}\cdot\frac{\cancel{L}}{3}\cdot\frac{w^3}{4} = \frac{Mw^2}{12}$$

You can check the solution from Table 9.2, case (d) and parallel axis theorem. The moment of inertia of thin rectengular plate, axis along edge, is given as

$$I_P = \frac{1}{3}Mw^2 = \frac{1}{12}Mw^2 + M\left(\frac{w}{2}\right)^2$$

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Section 5 **Quiz 9** **1 December 2011**

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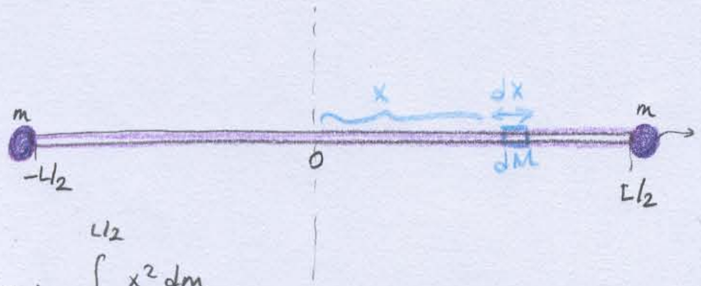
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

A rod of mass M and length L connects two small masses m. Find the moment of inertia of the composite system about an axis through the center, perpendicular to the rod.



small, point mass m $\Rightarrow I = m\left(\frac{L}{2}\right)^2$

$$I_{rod} = \int_{-L/2}^{L/2} x^2\, dm$$

$$\rho = \frac{M}{L} = \frac{dm}{dx} \Rightarrow dm = \frac{M}{L}\, dx$$

$$I_{rod} = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{M}{L}\cdot\frac{1}{3}\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] = \frac{ML^2}{12}$$

check from Table 9.2, case (a).

$$I_{total} = I_{rod} + 2 I_{mass} = \frac{ML^2}{12} + 2\cdot\frac{ML^2}{4} = \frac{7ML^2}{12}$$